

Wind-Based Robust Trajectory Optimization using Meteorological Ensemble Probabilistic Forecasts

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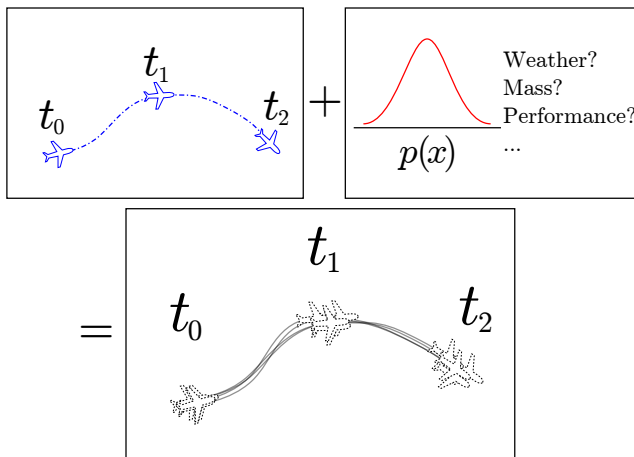
Area of Aerospace Engineering
Universidad Carlos III de Madrid

SESAR Innovation Days
TU Delft, 8th October 2016

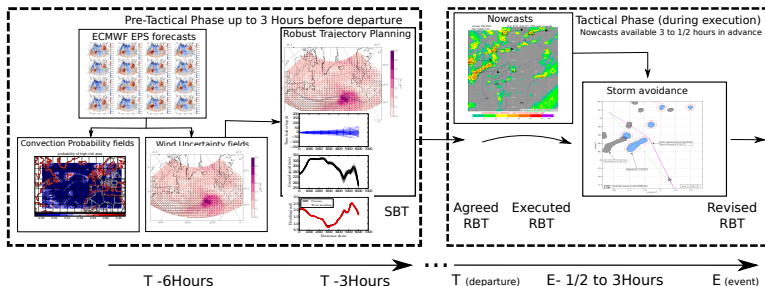
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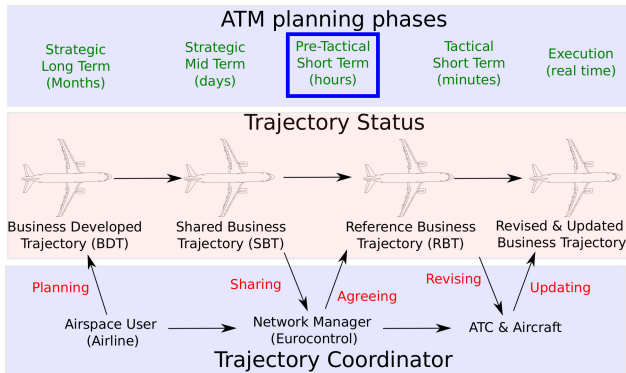
Uncertainty and 4D trajectories



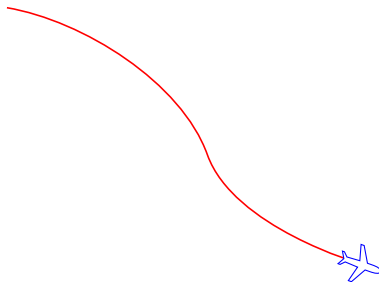
TBO-Met project



Research scope



4D Trajectory Planning



Contributions of this work

- Developing a methodology for 4D flight plan optimization under uncertainty
- Incorporate predictability into this methodology as an objective

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Why use ensemble forecasting?

Weather forecasts are uncertain, because nonlinear, complex, chaotic dynamics amplify:

- 1 Uncertainty in initial conditions
- 2 Physical modelling/parametrization errors
- 3 Numerical error and computational limitations

Deterministic forecasts have limited usefulness

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Ensemble Prediction Systems

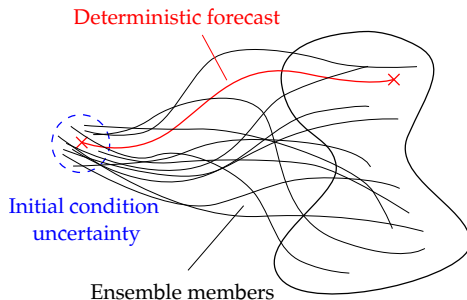


Figure: Uncertainty propagation

Ensemble Prediction Systems II

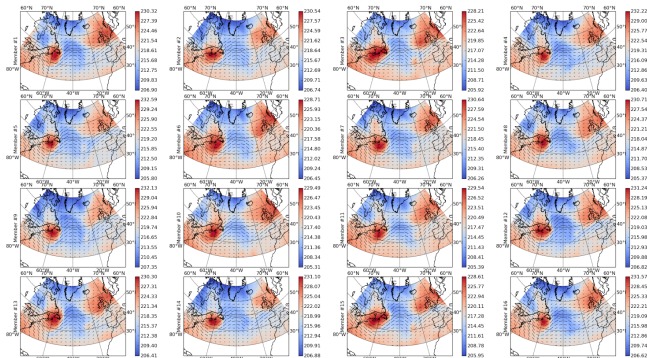


Figure: A subset of the PEARP ensemble forecast at 250 hPa

The 3-DoF Point-Mass Model

3 Position states:

Latitude (ϕ)

Longitude (λ)

Altitude (z)

1 Additional state:

Mass (m)

3 Velocity states:

Airspeed (v)

Heading (ψ)

Flight Path

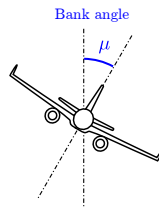
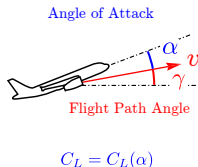
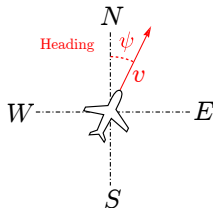
Angle (γ)

3 Controls:

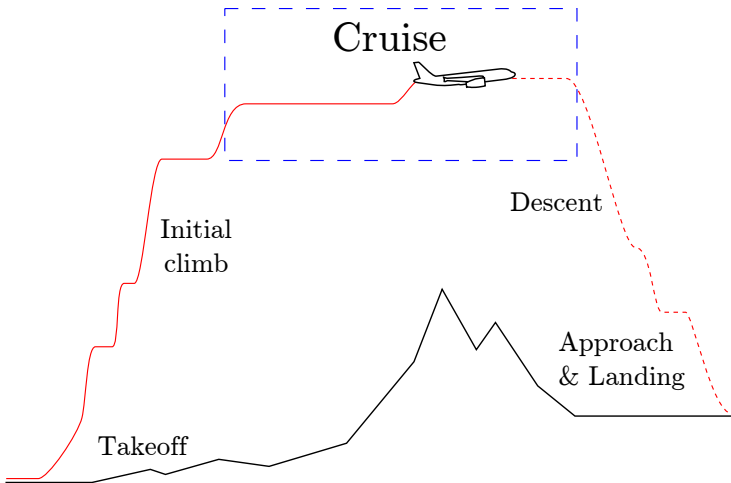
Throttle (π)

Lift Coeff. (C_L)

Bank Angle (μ)



Phases of a flight



Dynamic model for cruise

Cruise dynamical system

$$\dot{\phi} = \frac{v \cdot \cos(\psi) + w_y(\phi, \lambda, t)}{R_M + z}$$

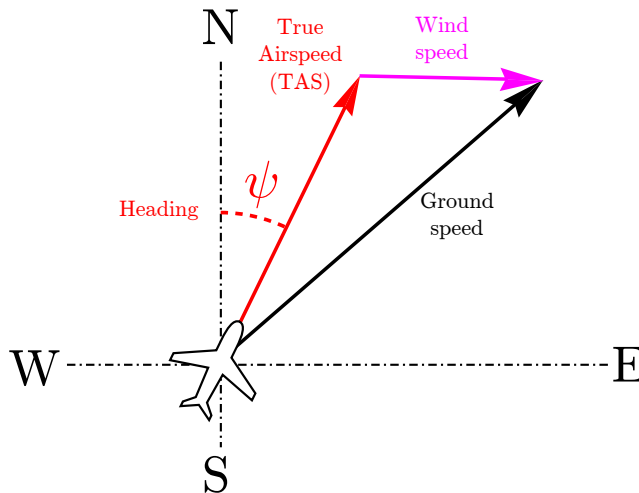
$$\dot{\lambda} = \frac{v \cdot \sin(\psi) + w_x(\phi, \lambda, t)}{(R_M + z) \cos(\phi)}$$

$$\dot{v} = m^{-1}(\text{Thrust}(\pi, v, \rho, T) - \text{Drag}(v, m, \rho, T))$$

$$\dot{m} = -\text{Fuel Consumption}(\pi, v, \rho, T)$$

- Forces, constraints & fuel consumption from BADA model

Wind impact on the dynamics



The Optimal Control Problem

OCP

$$x \in \mathbb{R}^n, u \in \mathbb{R}^m, g \in \mathbb{R}^{n_g}$$

$$\min J = \varphi(x_0, x_f, t_0, t_f) + \int_{t_0}^{t_f} \mathcal{L}(x, u, t) dt$$

subject to:

$$dx/dt = f(x, u, t)$$

$$g_L \leq g(x, u, t) \leq g_U$$

$$b_L \leq b(x_0, x_f, t_0, t_f) \leq b_U$$

- Common cost functional in ATM: $J = -m_f + Cl \cdot t_f$

Direct collocation methods

NLP

$$\min J = \varphi(x_0, x_M, t_0, t_M) + (t_f - t_0) \sum_{k=0}^M w_k \mathcal{L}(x_k, u_k, t_k)$$

subject to:

$$D_k(x_0, \dots, x_N) = f(x_k, u_k, t_k), k \in \{0, \dots, M\}$$

$$g_L \leq g(x_k, u_k, t_k) \leq g_U, k \in \{0, \dots, M\}$$

$$b_L \leq b(x_0, x_M, t_0, t_M) \leq b_U$$

The Dynamical System with Uncertainty

The dynamical system with uncertainty

$$dx/dt = f(x, u, t, p)$$

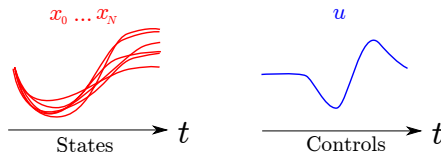
- Uncertainty is represented by a random variable $p \in \mathbb{R}^{n_p}$
- Note that the uncertainty p is static (i.e. this is not a SDE)

The robust OCP

- Discretize p with a Polynomial Chaos rule
- Approximate expectation with $\mathbb{E}(h(p)) = \sum_{i=0}^N w_i h(p_i)$
- Create a copy of the state x_i for each p_i and average the cost functional
- Problem size is $\mathcal{O}(nN)$

Dynamical system for the open-loop ROCP

$$dx_i/dt = f(x_i, u, t, p_i), \quad i \in \{0, \dots, N\}$$



The robust OCP II

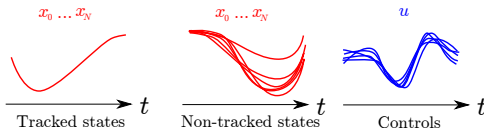
- Declare $q \leq m$ states to be tracked
- Make up to q controls scenario-specific

Dynamical system for the state-tracking ROCP

$$dx_i/dt = f(x_i, u, t, p_i), \quad i \in \{0, \dots, N\}$$

$$T(x_i - x_j) = 0$$

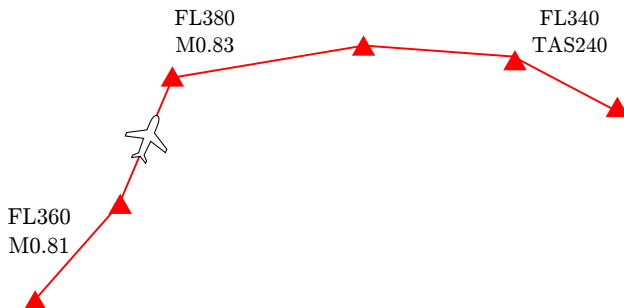
where $T \in \mathbb{R}^{n \times q}$ is the (full-rank) tracking matrix



Applying the ST-ROCP formulation to our problem

- EPS $\rightarrow$ already have a discretized p (with $w_i = N^{-1}$)
- Tracking of ϕ and λ is not practical because it forces to fully absorb uncertainty in v
- Not efficient
- Not operational
- Small feasible set (need to go slow to keep v margin for unfavourable scenarios)

Flight plans



Dynamic system reformulation

- Use ground distance (r) flown as independent variable. We can do this if $v_G > 0$ (true for realistic winds), as $r(t)$ is strictly increasing.
- Groundspeed is $v_G = \sqrt{(v \cos \psi + w_y)^2 + (v \sin \psi + w_x)^2}$.

Reformulated cruise dynamical system

$$dx/dr = \frac{\dot{x}}{v_G}$$

$$dt/dr = v_G(r)^{-1}$$

- Cost functional: $J = \mathbb{E}[-m_f + \text{CI} \cdot t_f] + \text{DP}(t_{f,\max} - t_{f,\min})$ with $t_{f,\min} \leq t_{f,i} \leq t_{f,\max}$

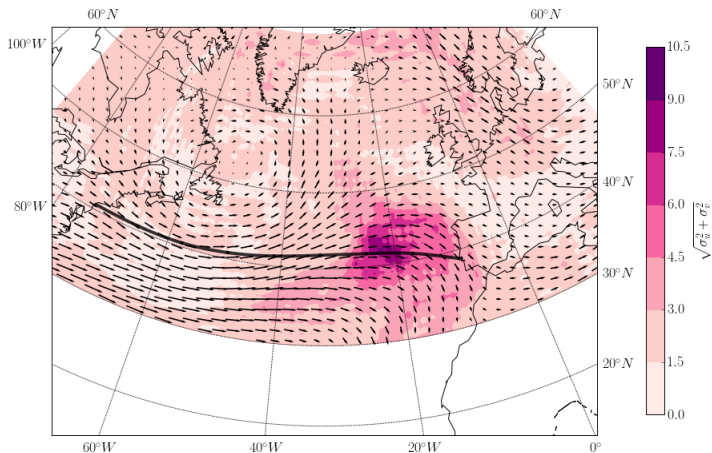
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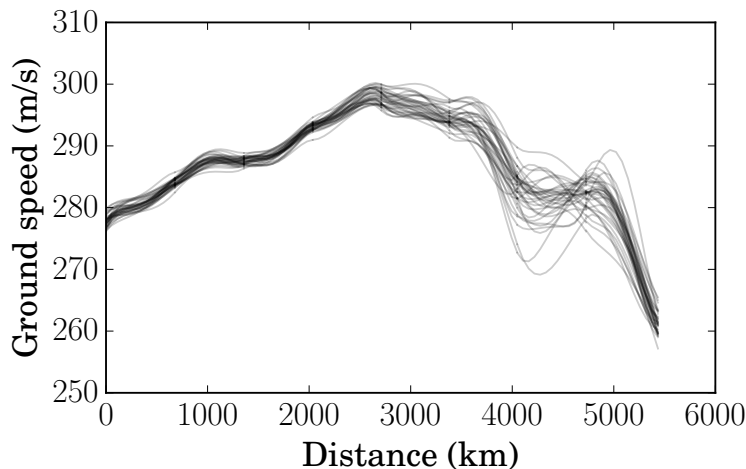
Description

- A330 from NY to Lisbon
- Flying at $M=.82$ and FL380
- 20th of January, 2016
- 200 hPa level forecast from the PEARP ensemble (35 members, from Météo France)

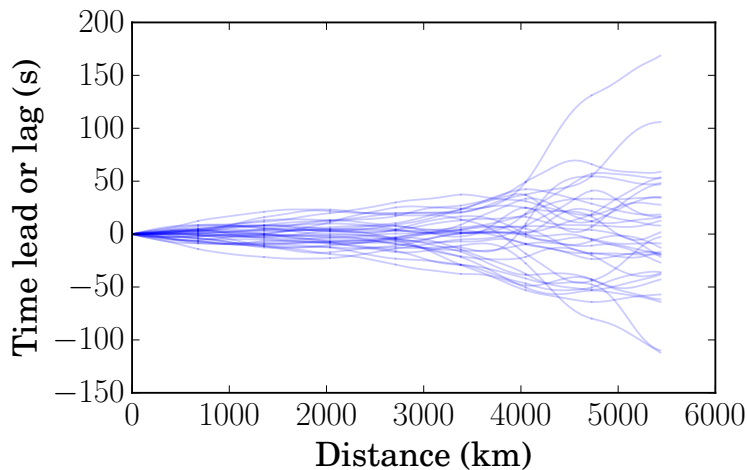
Routes



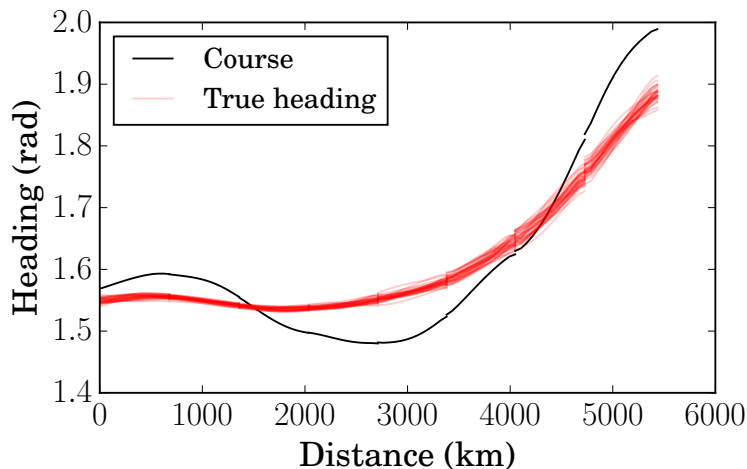
Groundspeed



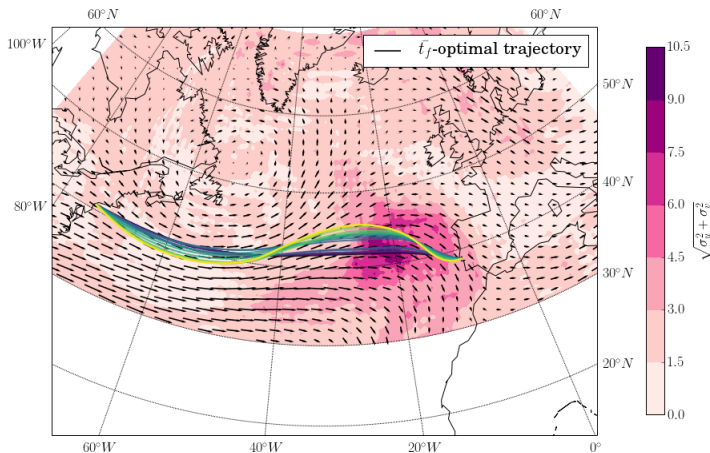
Time lead or lag



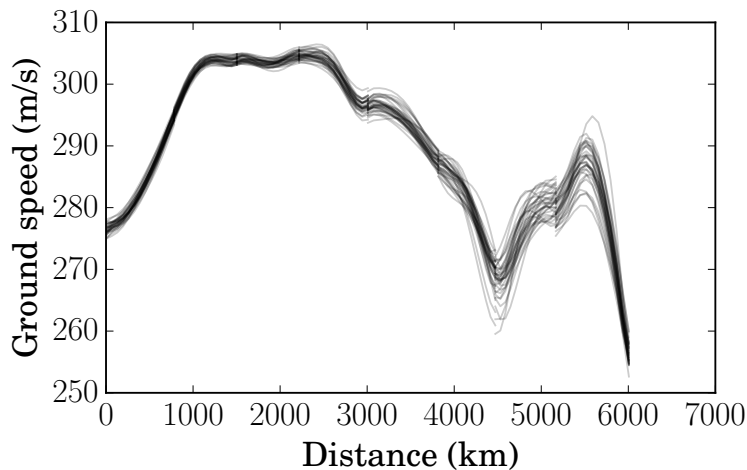
Time lead or lag



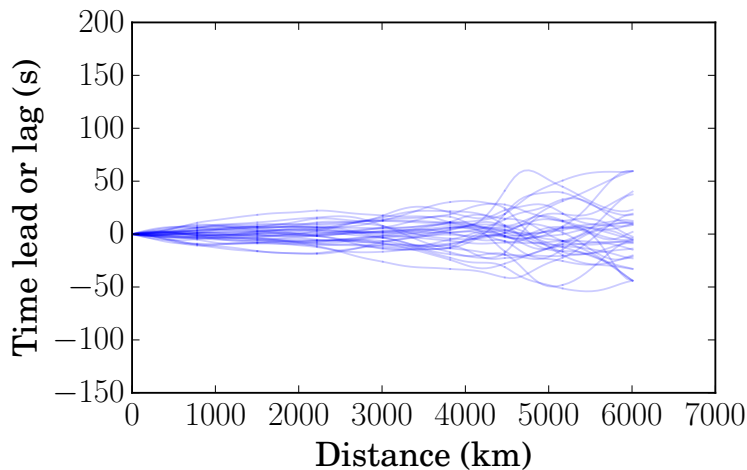
Routes



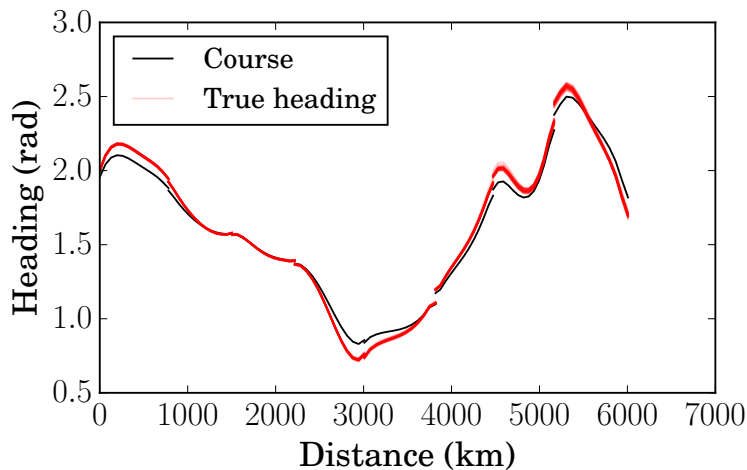
Groundspeed



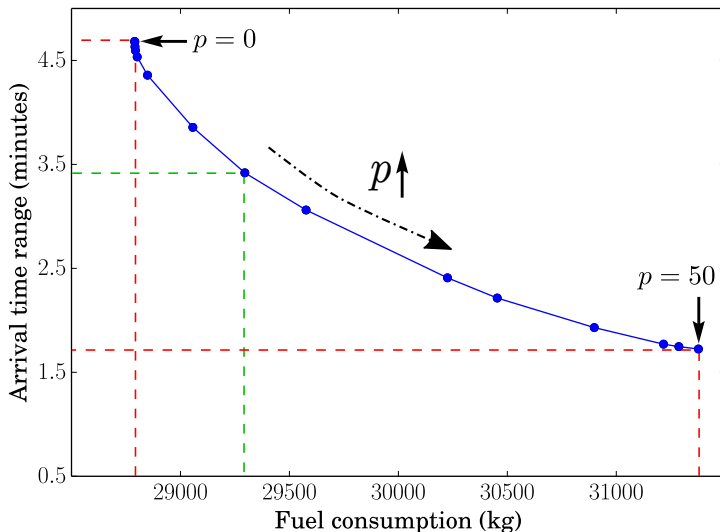
Time lead or lag



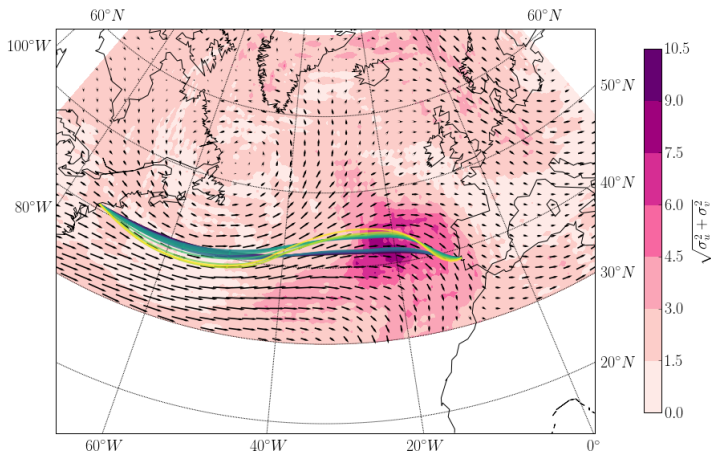
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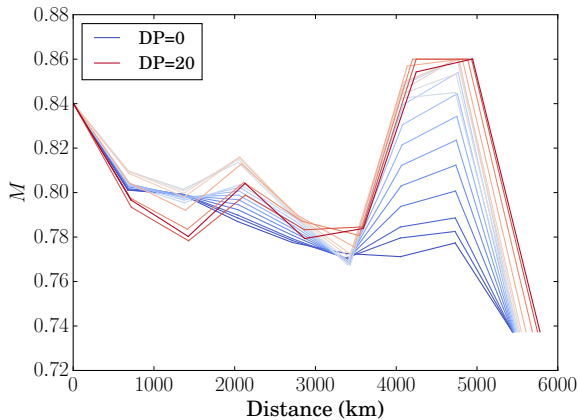
Predictability-efficiency trade-off



Routes (Variable velocity (CI=10))



Airspeeds (Cl=10)



Predictability-efficiency trade-off

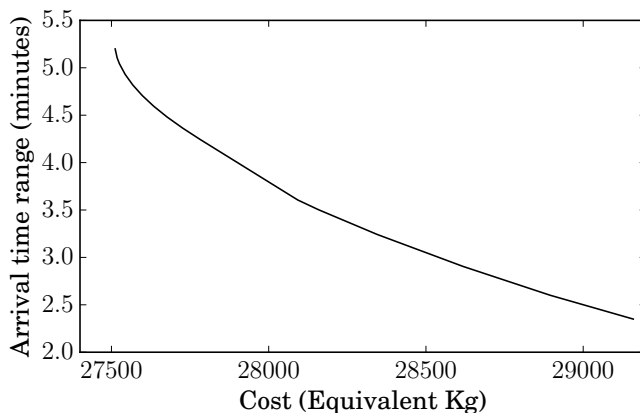


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Main takeaways

- We have developed a methodology to do flight planning under uncertainty and include predictability as an objective
- Adapted to operational concepts
- Scaling of $O(nN)$ is manageable (parallelization possibilities?)
- Same advantages and drawbacks of other direct methods (possibility of local optima but better handling of more complex, nonlinear and higher-dimensional systems compared to raw DP)
- Natural handling of EPS forecasts

Future work

- Dynamic Weather, BADA 4
- Extension to 4D (altitude changes)
- Assessment of potential benefits
- Combination with other uncertainty sources
- Extension to tactical avoidance of thunderstorms
- Conflict avoidance & resolution?

Acknowledgements

- HALA! network PhD scholarship
- TBO-Met project (check out poster!)

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