

Leveraging UAV Autonomy for Minimum 4D Flight Authorization Volumes

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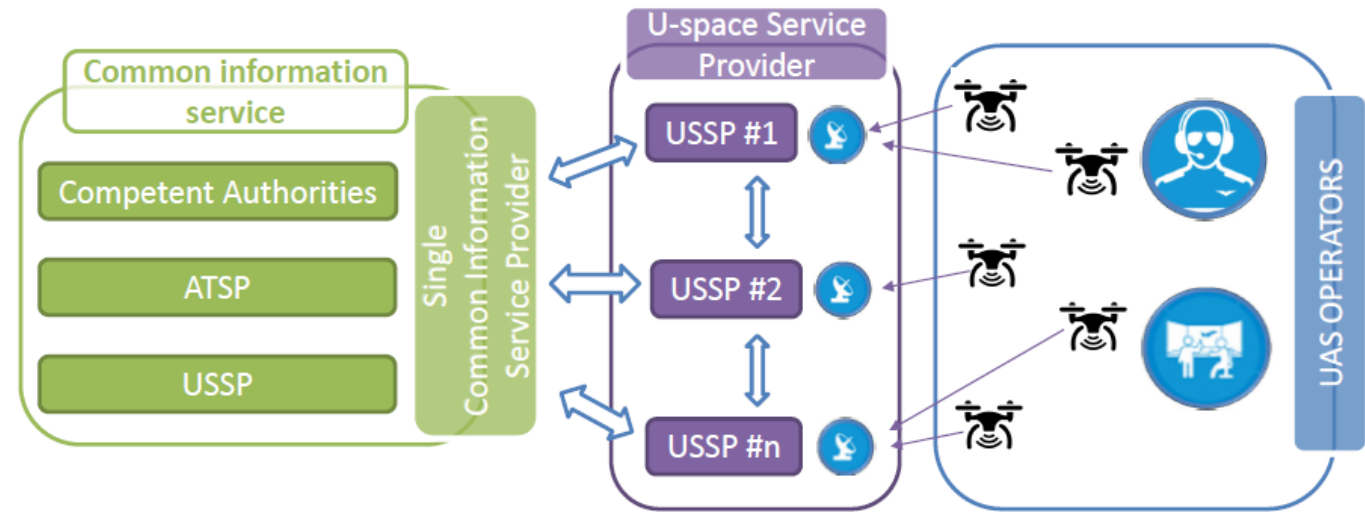
What is the U-space

- EASA's regulations (2021/664, 2021/665, 2021/666) on U-space came into force on the 26th January 2023
- The U-space is a geographical low-altitude airspace for drone flights that:
 - Enables BVLOS operations
 - Enables dense drone operations in a fair, safe, and efficient manner in urban environments
 - Enables co-existence between manned and unmanned aircrafts
 - Potentially unlock Innovative Air Mobility



U-space architecture

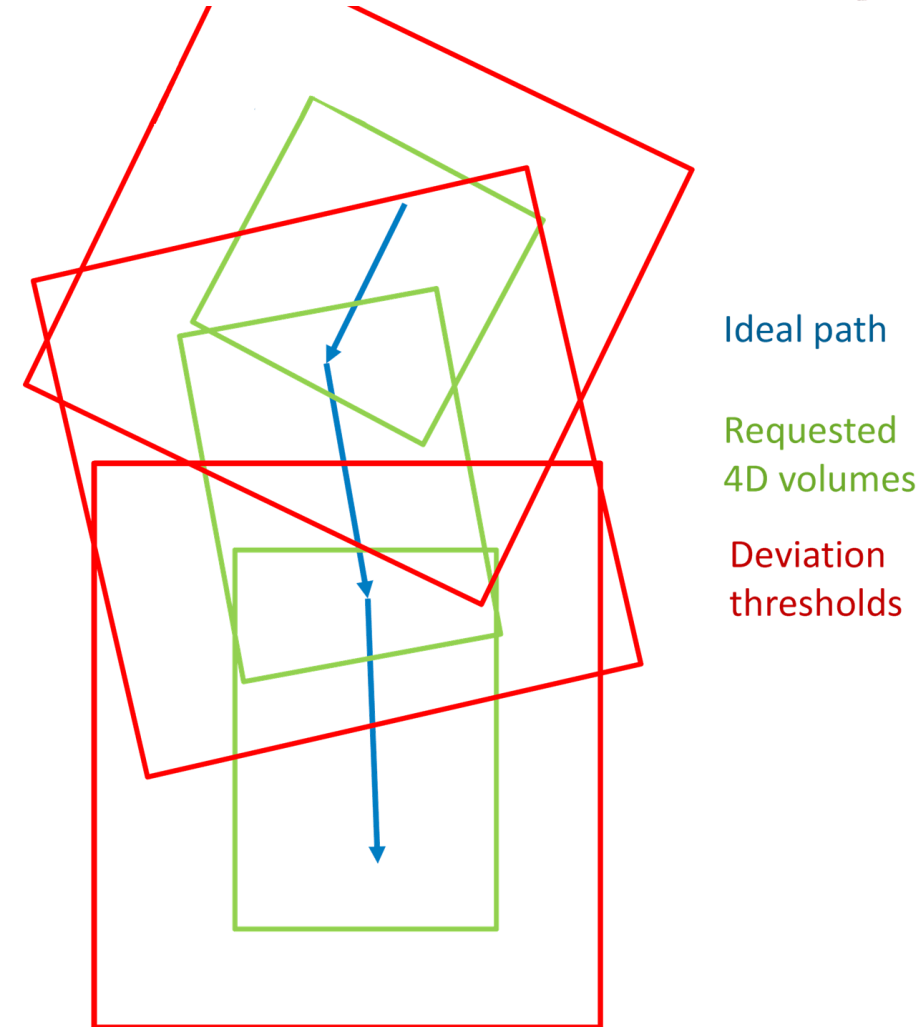
- Service based architecture
 - Geo-awareness service
 - Network Information
 - Traffic Information
 - Flight Authorization Service
 - Conformance Monitoring
 - Weather Service



The Flight Authorization service



- **Operator request:**
 - Operator submits a 4D trajectory reservation ($x, y, z + \text{time}$) aligned with the business/mission plan
 - The UAS must remain inside the reserved 4D volume with a defined probability margin
- **Service checks:**
 - Request is valid: timely, complete, from a registered operator
 - Compliance with airspace structure and capacity
 - Weather minima / maxima are respected
- **Deconfliction:**
 - A flight is authorized only if its 4D volume does not intersect any already authorized flight of equal or higher priority

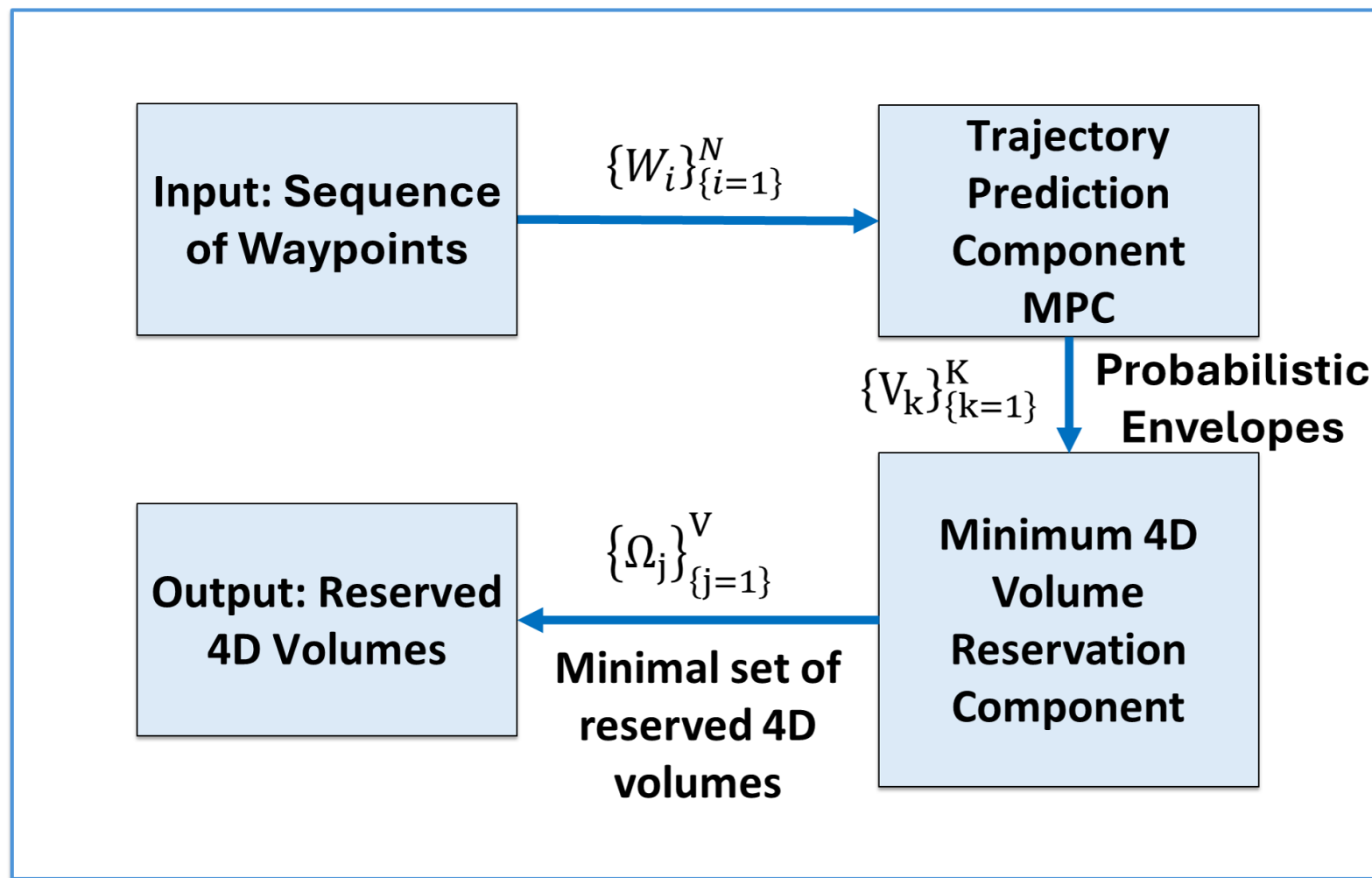




Objective: Optimized flight plans

- Today's flight authorizations:
 - oversized 4D volumes (space + time).
- Consequence: wasted airspace → fewer UAVs can fly simultaneously.
- Goal: exploit greater predictability of autonomous UAVs and reserve only what's needed, safely.

Overview of the approach



Trajectory predictions



- Waypoint

$$\mathbf{w}_i = \begin{bmatrix} \mathbf{x}_i^w \\ \dot{\mathbf{x}}_i^w \end{bmatrix} \in \mathbb{R}^6$$

- UAV state

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{p}_k \\ \dot{\mathbf{p}}_k \end{bmatrix} \in \mathbb{R}^6$$

- Motion model of the autonomous agent

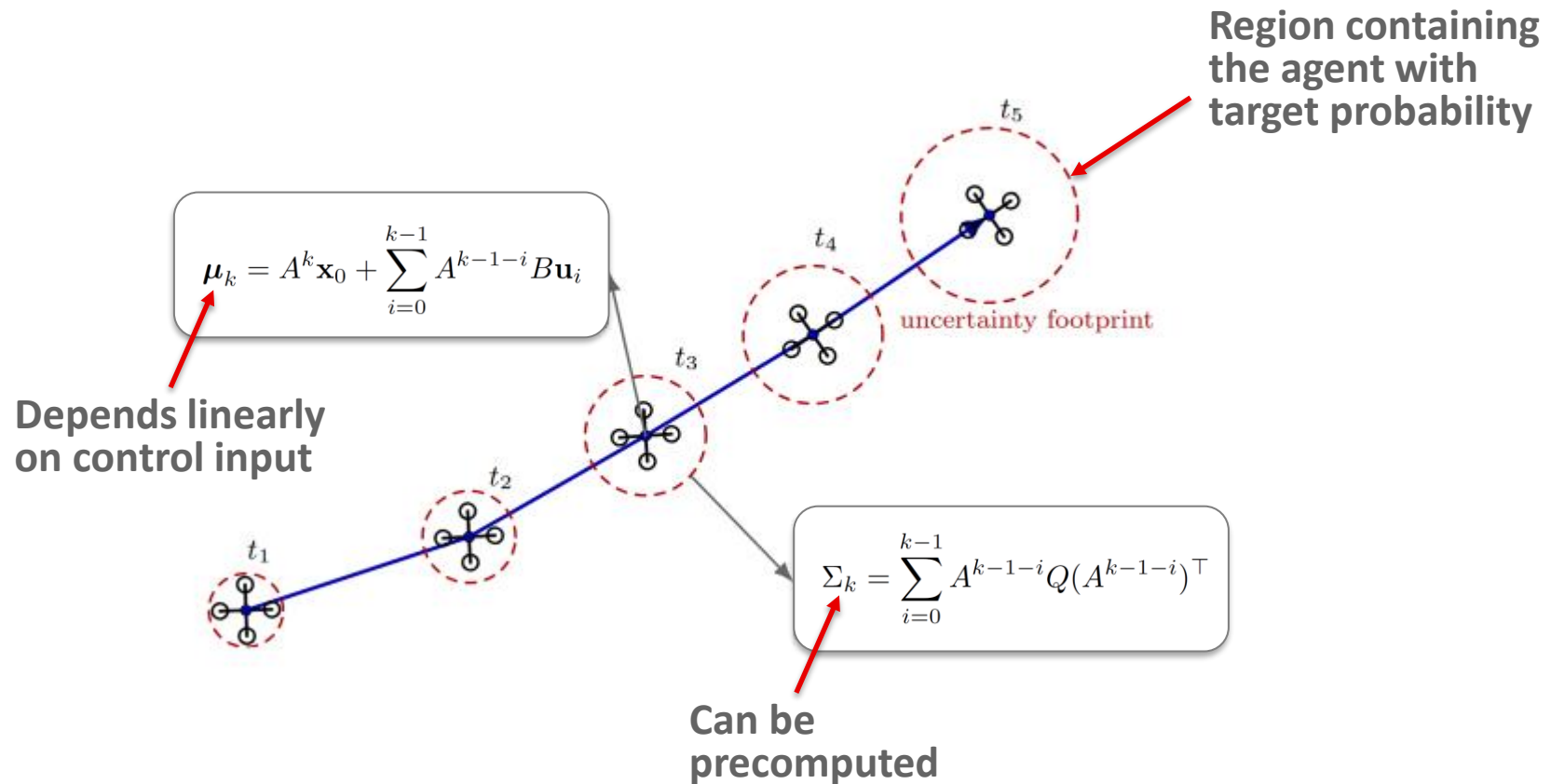
$$\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k + \mathbf{n}_k$$

Depends linearly on previous state Depends linearly on control input i.i.d. additive linear Gaussian noise

- Recursively predictive agent state

$$\mathbf{x}_k = A^k \mathbf{x}_0 + \sum_{i=0}^{k-1} A^{k-1-i} B \mathbf{u}_i + \sum_{i=0}^{k-1} A^{k-1-i} \mathbf{n}_i \quad \longrightarrow \quad \mathbf{x}_k \sim \mathcal{N}(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

Probabilistic envelopes



MPC-based control selection for optimal waypoint crossing



Large enough horizon

$$\min_{\{\mathbf{u}_k\}_{k=0}^{K_i-1}} \sum_{k=0}^{K_i} \rho^k \|\boldsymbol{\mu}_k - \mathbf{w}_i\|_2^2,$$
$$\text{s.t.} \quad \boldsymbol{\mu}_k = A^k \mathbf{x}_0 + \sum_{i=0}^{k-1} A^{k-1-i} B \mathbf{u}_i, \quad \forall k,$$
$$\|\mathbf{u}_k\|_\infty \leq u_{\max}, \quad \forall k,$$
$$\|\mathbf{u}_{k+1} - \mathbf{u}_k\|_\infty \leq \Delta u_{\max}, \quad \forall k,$$
$$\|\boldsymbol{\mu}_k^v\|_\infty \leq v_{\max}, \quad \forall k,$$

High-level Overview of Minimum Volume Flight Authorization Request



- After trajectory optimization, we obtain a probabilistic envelope $\mathcal{V}_k$ containing the agent at time k
- Minimum volume flight authorization optimization is now possible

$$\min_{\{\Omega_j, s_j, e_j\}_{j=1}^V} \sum_{j=1}^V \Omega_j \quad \leftarrow \text{4D volumes (space and time)}$$

$$\text{s.t.} \quad \mathcal{V}_k \subseteq \bigcup_{j=1}^V \Omega_j,$$

$$e_j - s_j \geq k^v,$$

Time duration of the 4D volume

$$e_j - s_{j+1} \geq k^o$$

Minimum time overlap between consecutive 4D volumes

Underlying Assumptions

- To keep the optimization tractable, we assume:
 - Flights operate up to the U-space ceiling
 - Only the x–y footprint matters (z is fixed)
 - This assumption can be relaxed if needed
 - Requested volumes are modeled as cuboid
 - Which reduce to rectangles in the x–y plane
 - With two allowable orientations (0° and 45°)



Min Volume Flight Authorization request

Problem Min Volume Reservation:

$$\min \sum_k \sum_j z_{kj} A_j$$

decision variables:

$$z_{kj} \in \{0, 1\} \quad \forall k, j$$

$$x_{\min}^j, x_{\max}^j, y_{\min}^j, y_{\max}^j, \quad \forall j$$

subject to:

$$x_{\min}^j \leq \mu_k^x - r_k + M(1 - z_{kj}) \quad \forall k, j$$

$$x_{\max}^j \geq \mu_k^x + r_k - M(1 - z_{kj}) \quad \forall k, j$$

$$y_{\min}^j \leq \mu_k^y - r_k + M(1 - z_{kj}) \quad \forall k, j$$

$$y_{\max}^j \geq \mu_k^y + r_k - M(1 - z_{kj}) \quad \forall k, j$$

$$A_j = w_j h_j \quad \forall j$$

$$w_j \geq x_{\max}^j - x_{\min}^j \quad \forall j$$

$$h_j \geq y_{\max}^j - y_{\min}^j \quad \forall j$$

$$\sum_j z_{kj} \geq 1 \quad \forall k$$

$$\sum_k z_{kj} \geq k^v \quad \forall j$$

$$s_j \leq k z_{kj} + M(1 - z_{kj}), \quad e_j \geq k z_{kj} \quad \forall k, j$$

$$\sum_k z_{kj} = e_j - s_j + 1 \quad \forall j$$

$$e_j \geq s_j, \quad s_{j+1} \geq s_j, \quad e_{j+1} \geq e_j \quad \forall j$$

$$e_j - s_{j+1} \geq k^o \quad \forall j$$

Envelope at time k
included in a cuboid

Usage of binary auxiliary variables

- A binary variable is introduced

- $z_{k,j}$ -> if volume j contains the agent envelope at time k , then $z_{k,j} = 1$

$$\sum_j z_{kj} \geq 1$$

$$x_{\min}^j \leq \mu_k^x - r_k + M(1 - z_{kj}) \quad \forall k, j$$

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4D Volume minimization

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Volumes are active at least k^v time steps

Subsequent volumes overlap at least k^o time steps

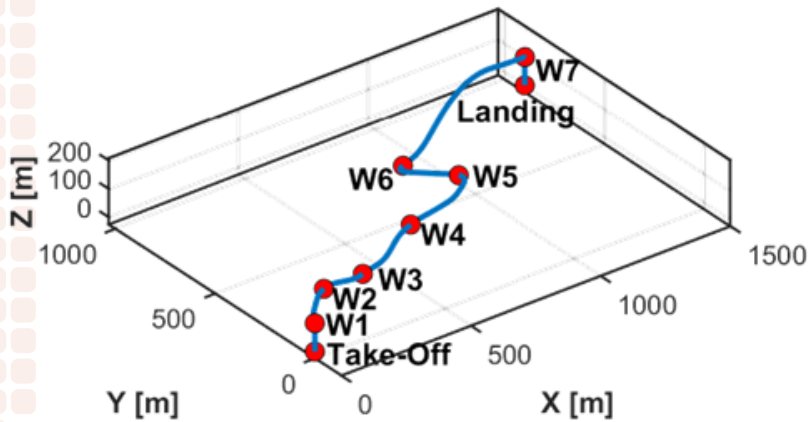


Discussion on the Min Volume Reservation optimization

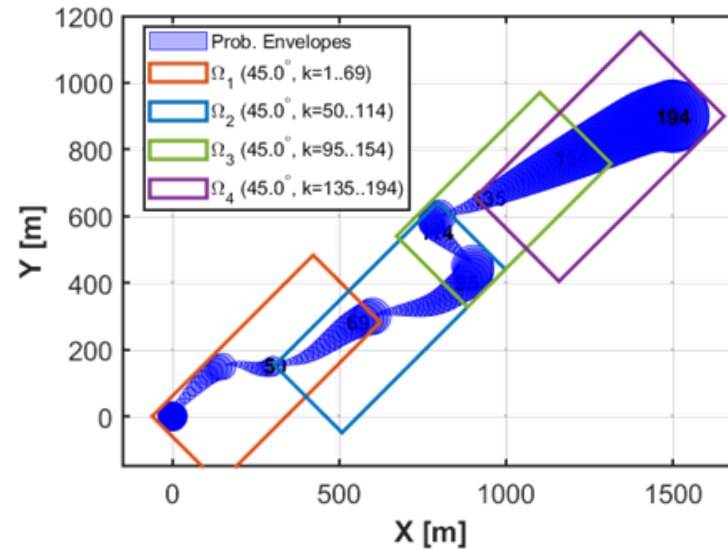


- The output of the optimization is the set of minimum volumes covering the agent envelopes while crossing through the desired waypoints
- The optimization is complex (mixed integer optimization), but is solved pre-flight, not in real-time
- Minimizing the 4D volumes enables the possibility to increase the number of agents in the airspace without reservation collision

Example application



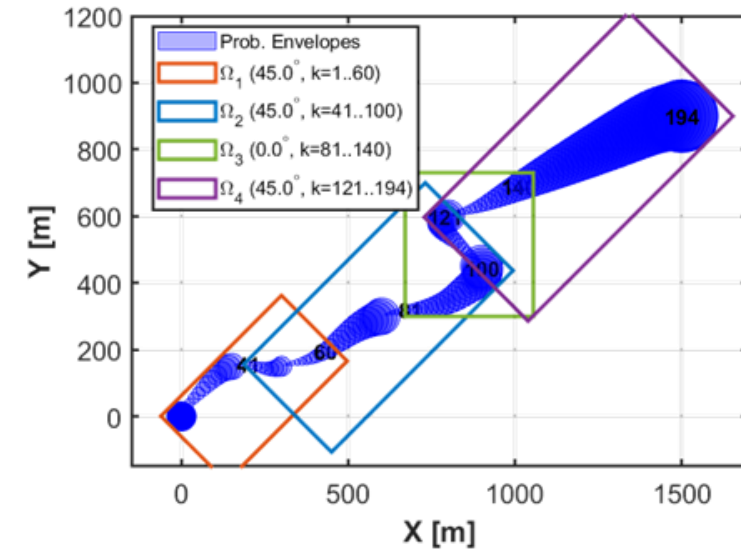
(a) Predicted UAV trajectory.



(b) Min Volume Reservation optimization.

$k^V = 60$ seconds

$k^O = 20$ seconds



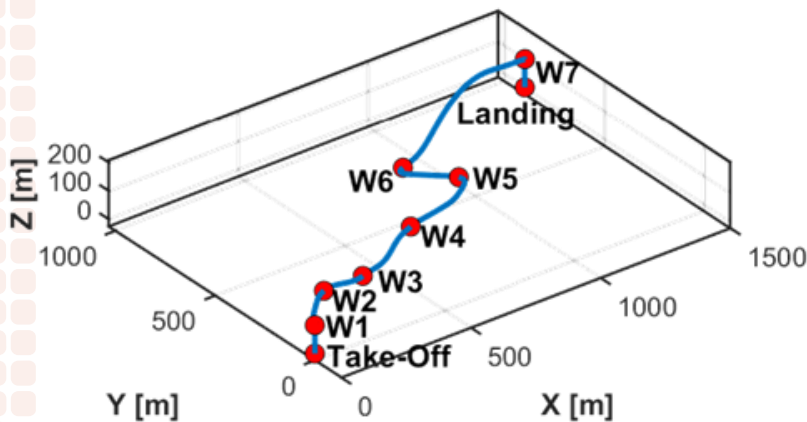
(c) Baseline approach.

Fixed Duration

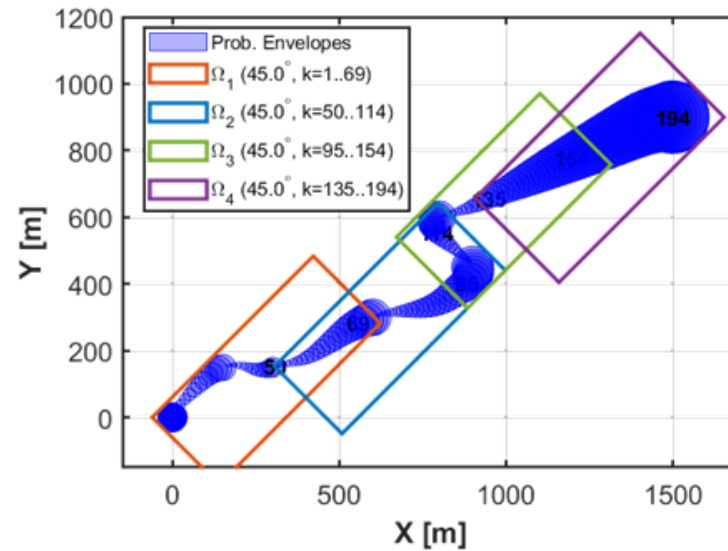
Fixed Overlap

Fixed Lateral volume

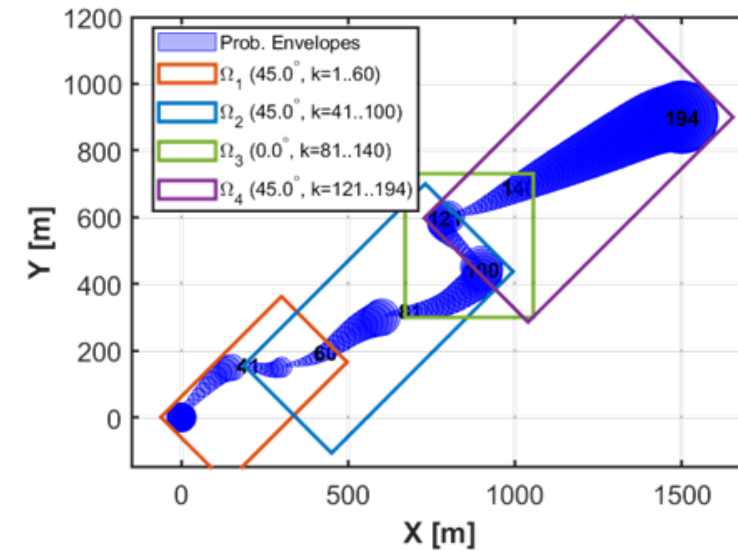
Example application



(a) Predicted UAV trajectory.



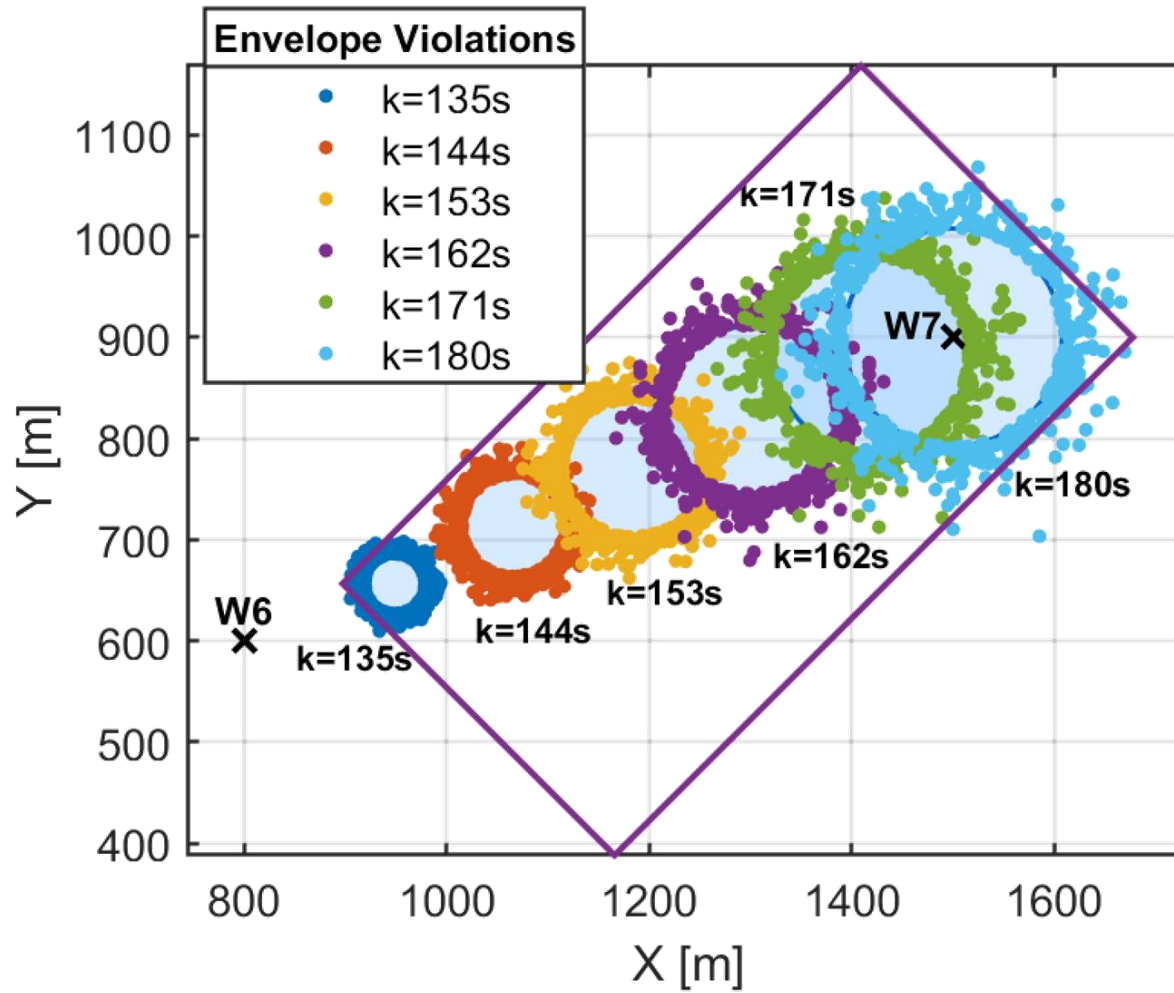
(b) Min Volume Reservation optimization.



(c) Baseline approach.

- Optimized 4D volumes computation requests 36% less airspace

Reserved Volume Robustness



- Probabilistic envelope confidence is respected (95%)
- 4D volume robustly cover the trajectories of the UAV
 - Only 0.08% of the trajectory have samples outside the reserved volume

Summary & Next Steps



- **The proposed flight authorization approach is:**
 - Efficient: more compact reservations → higher capacity.
 - Safe: probabilistic guarantees, compliance with U-space rules → higher capacity.
- **Next Steps**
 - Evaluate system-level capacity increases enabled by the framework
 - Assess framework flexibility
 - Incorporate ground-risk considerations
 - Integrate geo-fenced volumes

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**Funded by
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This project has received funding from the European Union's Horizon Europe research and innovation programme under grant agreement 101187121. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union. Neither the European Union nor the granting authority can be held responsible for them.

Thank You!

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